

## References for Roland Roeder’s 2023 MINT Mini-Course: “Dynamics of holomorphic and birational group actions on surfaces”.

Some nice general references on the automorphism group dynamics on K3 surfaces are the surveys [3, 6, 7]

### Lecture 1:

- (a) The affine surface  $S_D$  and its automorphisms: [5, 2, 15].
- (b) The smooth compact surfaces that were called  $W_A$  in my course appears on p. 1 of [12] and p. 3 of [3]. A more general discussion of the Wheeler surfaces appears in Section 3.1 of [4].
- (c) The dynamics on the space of pentagons is discussed in the introduction and Section 3.2 of [4].
- (d) The connection between the dynamics on character varieties and the automorphisms of  $S_D$  is described in [5, 2, 15]. See also the paper by Goldman [10] for more information on the Fricke-Klien trace relations and how they are used in character varieties.
- (e) The connection between the Painlevé 6 Monodromy and the dynamics on the affine surface  $S_D$  is discussed in [5, 2, 15].

### Lecture 2:

- (a) Proof that the canonical invariant volume form  $W_A$  is ergodic is a special case of Corollary 3.2 from [1] and Section 6.1 of [6] is a helpful alternative explanation.
- (b) The discussion about the dynamics of the automorphism group of  $S_D$  is from [15].

### Lecture 3:

- (a) This lecture was based entirely on further results on the dynamics of the automorphism group of  $S_D$  is from [15].
- (b) The Zassenhaus Lemma for Lie Groups can be found in Section 5.4 of [13]. A more classical reference is [14].
- (c) The “non-linear Zassenhaus Lemma” dates back to Ghys [9]. It plays an important role in [11] and the version presented here is Proposition 7.1 in [15]. It was recently used to study “canonical currents” in [8].

## REFERENCES

- [1] S. Cantat. Dynamique du groupe d’automorphismes des surfaces K3; *Transform. Groups* 6 (2001), no. 3, 201–214. See also: <https://perso.univ-rennes1.fr/serge.cantat/Articles/groupe.ps>
- [2] S. Cantat. Bers and Hénon, Painlevé and Schrödinger. *Duke Math. J.*, 149(3), 411–460, (2009). Long version from Cantat’s website: <https://perso.univ-rennes1.fr/serge.cantat/Articles/bhps-web.pdf>
- [3] Serge Cantat. Dynamics of automorphisms of compact complex surfaces. In *Frontiers in complex dynamics*, volume 51 of *Princeton Math. Ser.*, pages 463–514. Princeton Univ. Press, Princeton, NJ, 2014. See also: <https://perso.univ-rennes1.fr/serge.cantat/Articles/dyn-aut.pdf>
- [4] S. Cantat and R. Dujardin. Random dynamics on real and complex projective surfaces. Preprint: see long version at <https://perso.univ-rennes1.fr/serge.cantat/Articles/random1-web.pdf>.
- [5] S. Cantat and F. Loray. Dynamics on character varieties and Malgrange irreducibility of Painlevé VI equation. *Ann. Inst. Fourier (Grenoble)*, 59(7), 2927–2978, (2009). Long version from Cantat’s website: [https://perso.univ-rennes1.fr/serge.cantat/Articles/painleve\\_long.pdf](https://perso.univ-rennes1.fr/serge.cantat/Articles/painleve_long.pdf)

- [6] S. Filip. An introduction to K3 surfaces and their dynamics. Panoramas & Synthèses (2022) Teichmüller theory and dynamics, 1-43.  
See also [https://math.uchicago.edu/~sfilip/public\\_files/lectures\\_k3\\_dynamics.pdf](https://math.uchicago.edu/~sfilip/public_files/lectures_k3_dynamics.pdf)
- [7] S. Filip. Geometry and dynamics on Riemann and K3 surfaces. Eur. Math. Soc. Mag. 119 (2021), 17–22.  
<https://ems.press/journals/mag/articles/3718535>.
- [8] S. Filip and V. Tosatti. Gaps in the support of canonical currents on projective K3 surfaces.  
Preprint: <https://arxiv.org/abs/2302.08633>.
- [9] E. Ghys. Sur les groupes engendrés par des difféomorphismes proches de l'identité. *Bol. Soc. Brasil. Mat. (N.S.)*, 24(2), 137-178, (1993).
- [10] W. Goldman. Trace coordinates on Fricke spaces of some simple hyperbolic surfaces In: Handbook of Teichmüller theory. Vol. II, IRMA Lect. Math. Theor. Phys. 13, Eur. Math. Soc., Zrich, 2009. 611684.  
From arxiv: <https://arxiv.org/pdf/0901.1404.pdf>.
- [11] F. Loray and J. Rebelo. Minimal, rigid foliations by curves on  $\mathbb{C}\mathbb{P}^n$ . *J. Eur. Math. Soc. (JEMS)*, 5(2), 147-201, (2003).
- [12] C. McMullen. Dynamics on K3 surfaces: Salem numbers and Siegel disks *J. reine angew. Math.* 545(2002), 201—233. <https://people.math.harvard.edu/~ctm/papers/home/text/papers/k3/k3.pdf>
- [13] Lecture notes “Hyperbolic geometry and discrete groups” by Parreau and Will.  
<https://www.dpmms.cam.ac.uk/~aptm3/docs/lecture-notes/M2-HyperbolicGeometryAndDiscreteGroups.pdf>
- [14] Raghunathan, M.S. Discrete subgroups of semi-simple Lie groups. Springer-Verlag, New York 1972.
- [15] Julio Rebelo and Roland Roeder Dynamics of groups of birational automorphisms of cubic surfaces and Fatou/Julia decomposition for Painlevé 6. Preprint: <https://arxiv.org/pdf/2104.09256.pdf>